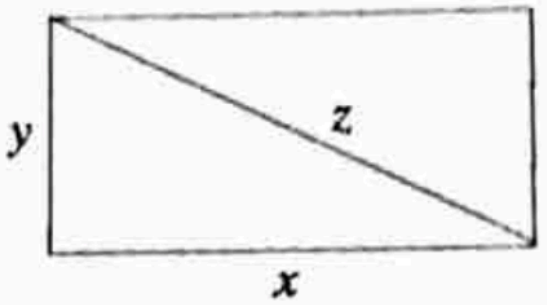
27. The diagonal z of the rectangle at the right is increasing at the rate of 2 cm/sec and $\frac{dy}{dt} = 3\frac{dx}{dt}$. At what rate is the length x increasing when $x = 3$ cm and $y = 4$ cm ?



(A) 1 cm/sec

(B) $\frac{3}{4}$ cm/sec

(C) $\frac{2}{3}$ cm/sec

(D) $\frac{1}{3}$ cm/sec

(E) $\frac{1}{15}$ cm/sec

27. Water is flowing into a spherical tank with 6 foot radius at the constant rate of 30π cu ft per hour. When the water is h feet deep, the volume of water in the tank is given by

$$V = \frac{\pi h^2}{3}(18 - h).$$

What is the rate at which the depth of the water in the tank is increasing at the moment when the water is 2 feet deep?

(A) 0.5 ft per hr

(B) 1.0 ft per hr

(C) 1.5 ft per hr

(D) 2.0 ft per hr

(E) 2.5 ft per hr

15. The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

(A) 5 in^3/sec

(B) 10 in^3/sec

(C) 15 in^3/sec

(D) 20 in^3/sec

(E) 25 in^3/sec

Ans

18. Let $y = 2e^{\cos x}$. Both x and y vary with time in such a way that y increases at the constant rate of 5 units per second. The rate at which x is changing when $x = \frac{\pi}{2}$ is

(A) 10 units/sec

(B) −10 units/sec

(C) −2.5 units/sec

(D) 2.5 units/sec

(E) −0.4 units/sec

Ans

10. When the area of an expanding square, in square units, is increasing three times as fast as its side is increasing, in linear units, the side is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 2 (E) 1

Ans

7. Administrators at Massachusetts General Hospital believe that the hospital's expenditures $E(B)$, measured in dollars, are a function of how many beds B are in use with

$$E(B) = 14000 + (B + 1)^2.$$

On the other hand, the number of beds B is a function of time t, measured in days, and it is estimated that

$$B(t) = 20 \sin\left(\frac{t}{10}\right) + 50.$$

At what rate are the expenditures decreasing when $t = 100$?

(A) 120 dollars/day

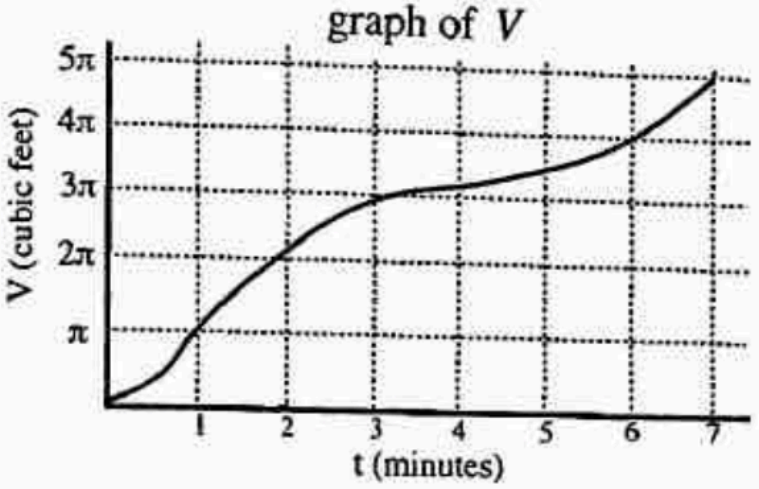
(B) 125 dollars/day

(C) 130 dollars/day

(D) 135 dollars/day

(E) 140 dollars/day

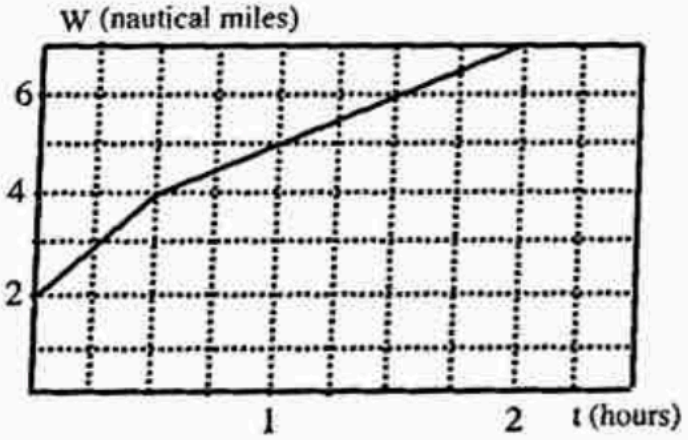
6. Sand is being dumped on a pile in such a way that it always forms a cone whose base radius is always 3 times its height. The function V whose graph is sketched in the figure gives the volume of the conical sand pile, $V(t)$, measured in cubic feet, after t minutes. $\left(V(t) = \frac{1}{3}\pi r^2 h\right)$ At what approximate rate is the radius of the base changing after 6 minutes?

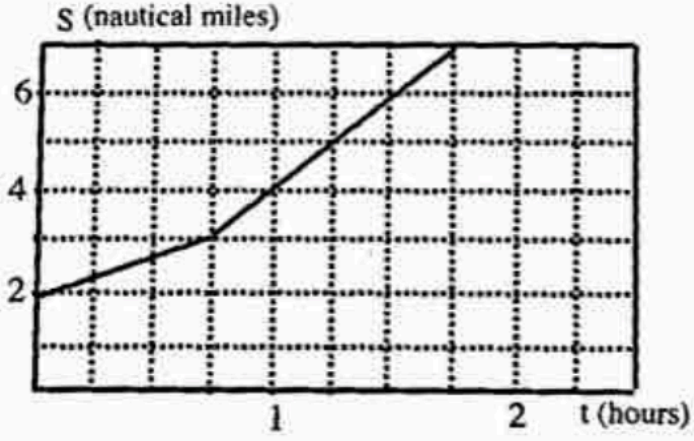


(A) 0.22 ft/min (B) 0.28 ft/min (C) 0.34 ft/min (D) 0.40 ft/min (E) 0.46 ft/min

Ans

12. One ship traveling west is $W(t)$ nautical miles west of a lighthouse and a second ship traveling south is $S(t)$ nautical miles south of the lighthouse at time t (hours). The graphs of W and S are shown below. At what approximate rate is the distance between the ships increasing at $t = 1$? (nautical miles per hour = knots)

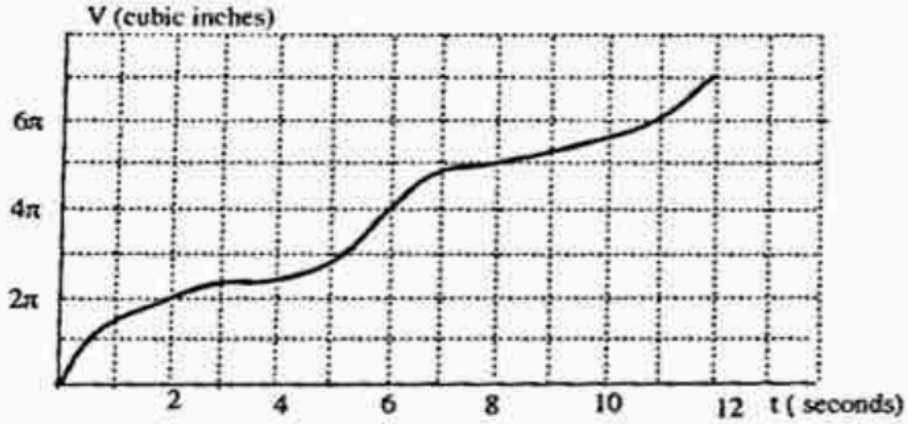




(A) 1 knot (B) 4 knots (C) 7 knots (D) 10 knots (E) 13 knots

12. The function V whose graph is sketched below gives the volume of air, $V(t)$, (measured in cubic inches) that a man has blown into a balloon after t seconds.

$$\left(V = \frac{4}{3}\pi r^3\right)$$



The rate at which the radius is changing after 6 seconds is nearest to

(A) 0.05 in/sec (B) 0.12 in/sec (C) 0.21 in/sec (D) 0.29 in/sec (E) 0.37 in/sec

Ans

3. Boyle's Law states that if the temperature of a gas remains constant, then the pressure P and the volume V of the gas satisfy the equation $PV = c$ where c is a constant. If the volume is decreasing at the rate of 10 in^3 per second, how fast is the pressure increasing when the pressure is 100 lb/in^2 and the volume is 20 in^3?

(A) $5 \frac{lb/in^2}{sec}$ (B) $10 \frac{lb/in^2}{sec}$ (C) $50 \frac{lb/in^2}{sec}$ (D) $200 \frac{lb/in^2}{sec}$ (E) $500 \frac{lb/in^2}{sec}$

Ans